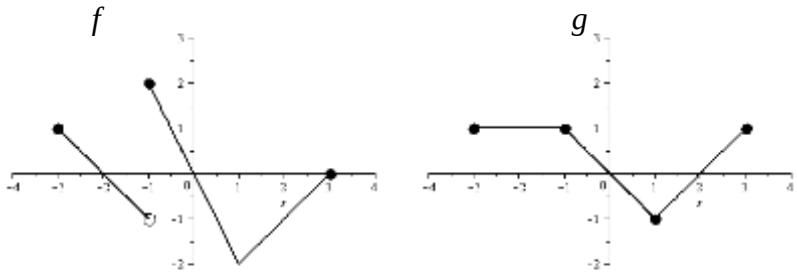
New Limits from Old
Homework Assignment

Use correct limit notation and words, where appropriate, to express your answers to the following problems.

1. Let *f* and *g* be the functions whose graphs are shown below. Use the graphs to evaluate the following limits. If a limit doesn't exist, explain why.

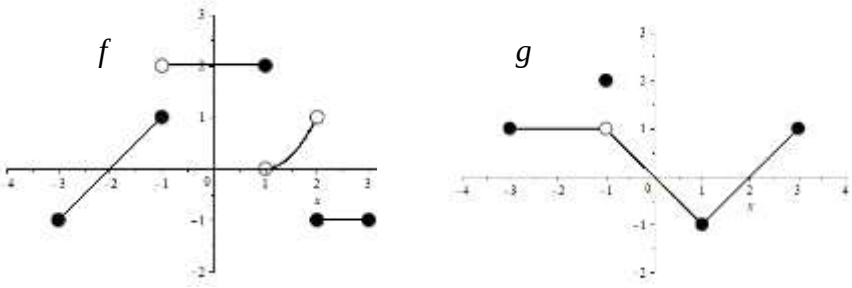


a. $\lim_{x\to 0} \frac{f(x)}{g(x)}$

b. $\lim_{x\to -3^+} \frac{f(x)}{g(x)}$

c. $\lim_{x\to 1} \frac{f(x)}{g(x)}$

2. Let *f* and *g* be the functions whose graphs are shown below. Use the graphs to evaluate the following limits. If a limit doesn't exist, explain why.



a. $\lim_{x\to 0}\big(f(x)+g(x)\big)$

b. $\lim_{x\to 2}\big(f(x)+g(x)\big)$

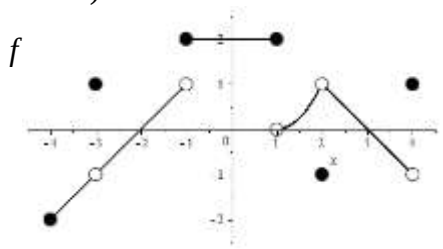
c. $\lim_{x\to 1}\big(f(x)g(x)\big)$

d. $\lim_{x\to 2}\big(f(x)g(x)\big)$

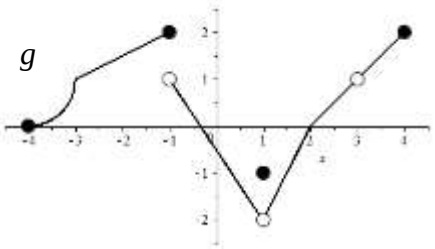
e. $\lim_{x\to -2} \frac{f(x)}{g(x)}$

f. $\lim_{x\to 0} \frac{g(x)}{f(x)}$

3. Use the graph of f to determine whether $\lim_{x\to 0}\dfrac{f(x)-2}{x}$ exists. If it does, compute it. If it doesn't, explain how you know. (Hint: it will help you to think geometrically about what this limit represents.)



4. Use the graph of g to determine whether $\lim_{h\to 0}\dfrac{g(2+h)}{h}$ exists. If it does, compute it. If it doesn't, explain how you know.



5. Graph the function $f(x)=|x|$ on the interval [-1,1]. Think carefully as you answer the following questions. (Hint: make of use the graph in setting up the difference quotients in the first two parts of the problem. It will simplify the problem!)
   a. First set up, then evaluate $\lim_{h\to 0^+}\dfrac{f(0+h)-f(0)}{h}$.
   b. First set up, then evaluate $\lim_{h\to 0^-}\dfrac{f(0+h)-f(0)}{h}$.
   c. Use this information to show that the function f is *not differentiable* at $x = 0$.
   d. Think about zooming in on the graph near zero. What do you see? Is the function locally linear at $x = 0$?

Note: You should see a connection between parts c. and d. of this problem. Do you? If not, you should ask about this!

6. Let $f(x)=\begin{cases} ax+1 & \text{if } x<2 \\ x^2 & \text{if } x\geq 2 \end{cases}$.
   a. Find the value of a for which $\lim_{x\to 2} f(x)$ exists.
   b. If a has the value found in part a, does $f'(2)$ exist? Justify your answer.